Automatic Damage Quantification Using Signal Based And Nonlinear Model Based Damage Sensitive Features

Meriem Ghrib [*] **Marc Rébillat** [*] **Nazih Mechbal** [*]
Guillaume Vermot des Roches [**]

[*] *PIMM laboratory, Arts et Métiers ParisTech, Paris, France*
(e-mail: meriem.ghrib@ensam.eu).
[**] *SDTools*
(e-mail: vermot@sdtools.com).

Abstract: Structural Health Monitoring (SHM) can be defined as the process of acquiring and analyzing data from on-board sensors to evaluate the health of a structure. Classically, an SHM process can be performed in four steps: detection, localization, classification and quantification. This paper addresses damage quantification issue as a classification problem whereby each class corresponds to a certain damage extent. Starting from the assumption that damage causes a structure to exhibit nonlinear response, we investigate whether the use of nonlinear model based features increases classification performance. A support Vector Machine (SVM) is used to perform multi-class classification task. Two types of features are used as inputs to the SVM algorithm: Signal Based Features (SBF) and Nonlinear Model Based Features (NMBF). SBF are rooted in a direct use of response signals and do not consider any underlying model of the test structure. NMBF are computed based on parallel Hammerstein models which are identified with an Exponential Sine Sweep (ESS) signal. A study of the sensitivity of classification performance to the noise contained in output signals is also conducted. Dimension reduction of features vector using Principal Component Analysis (PCA) is carried out in order to find out if it allows robustifying the quantification process suggested in this work. Simulation results on a cantilever beam with a bilinear torsion spring stiffness are considered for demonstration. Results show that by introducing NMBF, classification performance is improved. Furthermore, PCA allows for higher recognition rates while reducing features vector dimension. However, classifiers trained on NMBF or on principal components appear to be more sensitive to output noise than those trained on SBF.

Keywords: Damage quantification, Signal Based Features, Nonlinear Model Based features, SVM, PCA, output noise, cantilever beam, bilinear stiffness.

1. INTRODUCTION

Structural components used in mechanical, civil, and aerospace applications are often subjected to damage. Damage can lead to catastrophic structural failure if it is not identified in time. Therefore the implementation of SHM strategies and the development and exploitation of smart structures equipped with permanently attached sensing elements such as piezoelectric wafers are crucial. Generally, the damage monitoring process entails establishing: (1) the existence of damage, (2) the damage locations, (3) the types of damage, and (4) the damage severity (Rytter, 1993). Extensive research has been carried out to address the issues of damage detection and localization(Coverley and Staszewski, 2003; Vergé et al., 2010; Hajrya and Mechbal, 2013; Fendzi et al., 2016). However, very little research has been undertaken to respond to damage classification and quantification issues. In (Kim and Philen, 2011) damage classification is performed using time-frequency representations and the Adaboost machine learning algorithm. In (Mao and Todd, 2014), damage type classification is transformed into a group classification process, under the influence of uncertainty. More recently, Vitola *et al.* (Vitola et al., 2016) propose a data-driven methodology for the detection and classification of damages by using multivariate data driven approaches and machine learning algorithms.

These approcahes all have a common feature: they rely only on linear non model-based features as inputs to machine learning algorithms. But in many cases damage causes a structure to exhibit nonlinear response and the damage monitoring process can be significantly enhanced if one takes advantage of these nonlinear effects when extracting damage-sensitive features from measured data (Worden et al., 2008).

We thus aim here at exploiting a richer nonlinear representation of our test structure and at investigating whether the use of nonlinear model based features allows for an enhanced damage quantification approach. More specifically, the damage quantification problem is transformed into a classification problem whereby each class corresponds to a certain level of damage severity. A support vector machine is used to perform multiclass classification.

Signal Based Features (SBF) and Nonlinear Model Based Features (NMBF) are used to feed and train the SVM algorithm. SBF are based on a direct use of response signals and do not consider any underlying model of the structure under study. To compute NMBF, parallel Hammerstein models are considered to model the damaged structure. The model is identified using an Exponential Sine Sweep (ESS) excitation signal and NMBF are afterwards computed based on the identified Hammerstein kernels. PCA has generally been used in SHM field as a technique to establish damage sensitive features (Hajrya and Mechbal, 2013; Tibaduiza et al., 2013). In this work PCA is used to reduce the dimension of features vector, the aim being to find out if dimension reduction allows robustifying the suggested quantification approach. Furthermore a study of the sensitivity of classification performance to the noise contained in output signals is performed. Simulation results on a realistic cantilever beam with a bilinear torsion spring stiffness are considered as a demonstration example.

The remaining of the paper is organized as follows: Firstly, the test structure considered in this work is presented. Then, the main key ingredients of the proposed quantification worflow are introduced. Simulation results used to derive damage sensitive features are afterwards described. Results and analyses are presented thereafter. Conclusions and perspectives are finally drawn.

2. TEST STRUCTURE

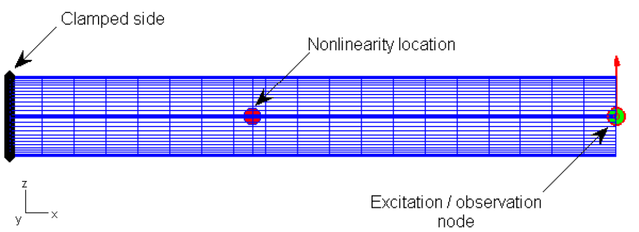

Fig. 1. Illustration of the simulated model

The test structure on which we conducted our study is a simulated beam model developed in a previous work for the investigation of a vibration-based structural health monitoring procedure on ceramic insulators (Rébillat et al., 2014). SDTools Matlab toolbox (SDT for Matlab) (Balmès, 2016) is used to simulate the dynamic reponse of our model. The base model is a cantilever beam of length $4m$ and circular section of radius $0.17m$. At a given nodal position a torsion spring is placed between the two rotational Degree Of Freedom (DOF) instead of the continuum coupling. The torsion spring stiffness (parameter k_v) is calibrated to be close to its saturation level, the threshold is defined as 95% of the maximum frequency of the first mode. The torsion spring has a bilinear behaviour acting as a nonlinearity in the system. The nonlinearity is defined as relative to a healthy state for which the torsion spring behaves linearly. One defines a generic damage severity parameter α that should vary between 0 (healthy) and 1 (fully damaged). The bilinear stiffness physically corresponds to a crack that is opening and closing and thus applying a lower stiffness in traction. Compression stiffness is set to k_v. Traction stiffness is set to $(1-\alpha)k_v$.

The relation between force (f_s) and displacement (Δx) for this element is given by : (1).

$$f_s(\Delta x) = \begin{cases} k_v \Delta x & \text{if } \Delta x < 0 \\ (1-\alpha)k_v\Delta x & \text{if } \Delta x \geq 0 \end{cases} \tag{1}$$

The excitation is a punctual force in the z direction (red arrow on figure 1) whose amplitude is defined by an exponential sine sweep curve. Various parameters can be specified to define the sweep signal. These parameters include f_{min} (the minimum frequency), f_{max} (the maximum frequency), f_s (the sampling frequency), t_{length} (the signal duration) and amp (the signal amplitude). The location of the excitation is specified by parameter $inpos$. The observation is a nodal translation response whose position is defined by parameter $outpos$.

3. QUANTIFICATION APPROACH

Figure 2 illustrates the main key ingredients of the quantification workflow proposed in this work. An input signal is firstly selected to excite a test structure containing a certain damage severity. The structure response signal is then recorded and damage sensitive features are extracted. In this work, the first question which arises is whether NMBF allows for an enhanced damage quantification strategy. Two types of features are thus considered: SBF and NMBF.

3.1 Signal Based Features: SBF

Signal Based Features are rooted in a direct use of response signals and do not consider any underlying model of the test structure. Four signal based features are considered in this study and are computed as follows. Let $s_{ref}(t)$ and $s_d(t)$ be the structure output signal in reference and damaged state respectively, where t refers to time, we define:

- **Correlation Coefficient damage index**
$$CC = 1 - \frac{cov(s_{ref}(t), s_d(t))}{\sigma_{s_{ref}(t)}\sigma_{s_d(t)}} \tag{2}$$
where $cov(s_{ref}(t), s_d(t))$ is the covariance of $s_{ref}(t)$ and $s_d(t)$, $\sigma_{s_{ref}(t)}$ and $\sigma_{s_d(t)}$ are the standard deviations of $s_{ref}(t)$ and $s_d(t)$ respectively.

- **Normalized Residual Energy**
$$NRE = \frac{\sum_{t=T_1}^{T_2}(s_{ref}(t) - s_d(t))^2}{\sum_{t=T_1}^{T_2} s_{ref}(t)^2} \tag{3}$$
where $[T_1, T_2]$ is the time interval in which signals of interest are analyzed.

- **Maximum Amplitude**
$$MA = \frac{\max_t(|s_{ref}(t) - s_d(t)|)}{\max_t |s_{ref}(t)|} \tag{4}$$

- **Signal envelope or instant amplitude energy**
$$ENV = \sqrt{\frac{\sum_{t=T_1}^{T_2} A^2_{s_{ref,d}(t)}}{\sum_{t=T_1}^{T_2} A^2_{s_{ref}(t)}}} \tag{5}$$

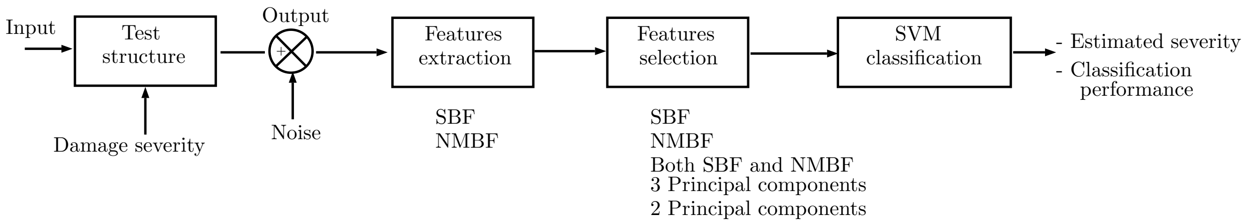

Fig. 2. Workflow suggested for damage quantification

where $s_{ref,d}(t) = s_{ref}(t) - s_d(t)$
$A_{s(t)} = \sqrt{s^2(t) + \mathcal{H}\{s\}(t)^2}$
$\mathcal{H}\{s\}(t)$ is the Hilbert transform of $s(t)$

Readers who are interested in more details about the signal based features considered herein are directed to (Fendzi, 2015).

3.2 Nonlinear Model Based Features: NMBF

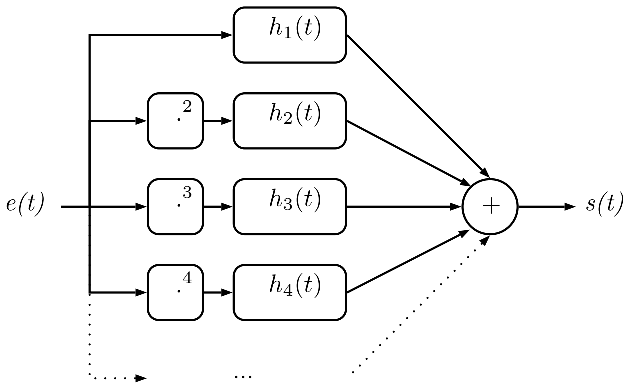

Fig. 3. Parallel Hammerstein models

NMBF are considered based on previous work presented in (Bakir et al., 2015; Rébillat et al., 2013). Parallel Hammerstein models are used to model the damaged structure (see Figure 3). The model is fully represented by its kernels $\{h_n(t)\}_{n \in \{1 \dots N\}}$ (N being the model order and can be automatically estimated (Rébillat et al., 2016)). The model is identified by means of Exponential Sine Sweeps excitation signal ($e(t)$). The system output $s(t)$ can be rewritten as follows:

$$s(t) = \sum_{n=1}^{N} (h_n * e^n)(t) \tag{6}$$
$$= (h_1 * e)(t) + \sum_{n=2}^{N} (h_n * e^n)(t) \tag{7}$$
$$= s^L(t) + s^{NL}(t) \tag{8}$$
$$= s^L(t) + \sum_{n\ odd} (h_n * e^n)(t) + \sum_{n\ even} (h_n * e^n)(t) \tag{9}$$
$$= s^L(t) + s_o^{NL}(t) + s_e^{NL}(t) \tag{10}$$

where the output signal is decomposed into a linear part and a nonlinear part (7). The nonlinear part is in turn decomposed into odd harmonics contribution and even harmonics contribution (9). Three features are chosen and computed as detailed hereafter.

- **Frequency shift**

$$f_{shift} = \frac{f_d - f_{ref}}{f_{ref}} \tag{11}$$

where f_d is the frequency of the first mode of the structure in the damaged case,
f_{ref} is the frequency of the first mode of the structure in the reference case.
These frequencies can here be easily extracted from the estimated nonlinear model as the kernel $h_1(t)$ corresponds to the linear response of the system.

- **Ratio of the nonlinear energy to the linear energy: *NLL***

$$NLL = \frac{\int_{f_1}^{f_2} |S^{NL}(f)|^2 \,\mathrm{d}f}{\int_{f_1}^{f_2} |S^{L}(f)|^2 \,\mathrm{d}f} \tag{12}$$

where $S^{NL}(f)$ is the nonlinear part of the system output in the frequency domain,
$S^L(f)$ is the linear part of the system output in the frequency domain,
$[f_1, f_2]$ is the frequency interval in which signals of interest are analyzed.

- **Ratio of the even to the odd nonlinear energies *EO***

$$EO = \frac{\int_{f_1}^{f_2} |S_e^{NL}(f)|^2 \,\mathrm{d}f}{\int_{f_1}^{f_2} |S_o^{NL}(f)|^2 \,\mathrm{d}f} \tag{13}$$

where $S_e^{NL}(f)$ corresponds to even harmonics contribution to the nonlinear part of the system output in the frequency domain,
$S_o^{NL}(f)$ corresponds to odd harmonics contribution to the nonlinear part of the system output in the frequency domain.

3.3 SVMs and PCA

SVMs SVM learning technique is used for the classification step. SVMs (Cristianini and Shawe-Taylor, 2000) are originally introduced by Vapnick and co-workers (Boser et al., 1992; Vapnik, 1998) and successfully extended by a number of other researchers. SVMs are applicable to both classification and regression. When used for classification, SVMs separate a given set of binary labeled training data with a hyper-plane that is maximally distant from them (known as the maximal margin hyper-plane). For cases in which no linear separation is possible, they can work in combination with the technique of 'kernels', that automatically realizes a non-linear mapping to a feature space. the hyper-plane found by the SVM in the feature space correponds to a non-linear decision boundary in the input space. To extend SVMs to multi-class scenario, a typical conventional way is to decompose a multi-class problem into a series of two-class problems. One can distinguish between two implementations:

- One Against All 'OAA' approach
- One Against One 'OAO' approach

The 'OAO' and the 'OAA' are two popular strategies for multi-class SVM. 'OAO' builds one SVM for each pair of classes while 'OAA' consists of building one SVM per class, trained to distinguish the samples in a single class from the samples in all remaining classes. In this work, a Gaussian kernel SVM is considered. SVM and Kernel Methods (SVM-KM) matlab toolbox (Canu et al., 2005) is used to perform multiclass classification.

PCA Principal component analysis (PCA) (Jolliffe, 1986) is a popular tool for linear dimensionality reduction and feature extraction. Intuitively, PCA can supply the user with a lower-dimensional picture of data when viewed from its most informative viewpoint. Several extentions of the standard PCA have been proposed such as the Kernel PCA which is the nonlinear form of PCA and which better exploits the complicated spatial structure of high-dimensional features. In this work we opted for the standard PCA since our features vector is not very high-dimensional.

3.4 Features selection

The step of features selection deals with the issue of which features to select to feed and train the SVM algorithm. In this work, five scenarios are tested:

- **Scenario 1** Only SBF are used to train the SVM algorithm
- **Scenario 2** Only NMBF are used to train the SVM algorithm
- **Scenario 3** Both SBF and NMBF are used to train the SVM algorithm
- **Scenario 4** PCA is performed on both SBF and NMBF and only 2 principal components, which account for 84% of data variance, are used to train the SVM algorithm
- **Scenario 5** PCA is performed on both SBF and NMBF and only 3 principal components, which account for 98% of data variance, are used to train the SVM algorithm

4. MODEL SIMULATION AND FEATURES DATABASE

The steps presented hereafter provides the high level calls employed to obtain simulation results, damage sensitive features, classification results as well as classification performances.

(1) **Initialize parameters**
(2) **Run simulations**
(3) **Add noise to output signals**
(4) **Compute and save SBF**
(5) **Compute and save NMBF**
(6) **Learn and test classifiers**
(7) **Compare classifiers performances**

Concerning step (2), simulations are done with damage severities varying from 0 to 0.9 with steps of 0.1. Each damage severity is considered at damage locations $NLpos$ varying from 0.1 to 0.9 with steps of 0.1. Setting $NLpos$ to a given value $v, v \in [0, 1]$, stands for a damage position at $v \times 100\%$ of the beam length from the clamped side. The localization of the excitation force is set to 1, standing for 100% of the beam length from the clamped side. The position of the observation node is set at the excitation localization. After running simulations 90 output signals are measured.
Concerning step (3), white Gaussian noise with SNR (14) varying from 20 dB to 100 dB by steps of 5 dB is added to the output signals.

$$SNR_{dB} = 10 \log_{10}(\frac{P_{signal}}{P_{noise}}) \tag{14}$$

where P_{signal} and P_{noise} are signal and noise power respectively.
In order to enrich our features database, 10 repetitions were considered for each SNR value. With such parameters, a total of 15300 noisy output signals are obtained: 1530 noisy output by each class of damage severity. The output signals obtained at this step are used to compute damage sensitive features (step (4) and (5)). Thus, one obtains 1530 samples by each class of damage severity. Finally for step (6), various classifiers are trained according to SVM input features scenarios (See section 2). For training, only damage sensitive features computed using output signals with a maximum SNR ($SNR_{max} = 100dB$) are used. We assume that for training we consider the most favourable case where noise is very low. For real applications this may correspond to a learning via models or in well-controlled environments. For test we consider less favourable situations where output noise is not negelected. Thus features computed using output signals with a SNR lower than SNR_{max} are used to test the classifiers.

5. RESULTS AND ANALYSES

5.1 Simulation results

Figure 4 plots output signals amplitude as a function of damage severity for a damage location of 0.8. It can be seen that an increase in damage severity results in greater distortions of output signals. The same trend was observed for the various considered damage positions.

5.2 SBF as function of damage severity

From figure 5, it can be seen that SBF (CC, NRE, MA and ENV) increase monotonically with damage severity. Such trend is observed for the various SNRs considered in this work. Furthermore, it is worth noting that similar results are obtained for all damage positions considered in this investigation.

5.3 NMBF as function of damage severity

From figure 6, it can be seen that the first two NMBF (f_{shift} and NLL) increase monotonically with damage severity. The third NMBF (EO) does not show a

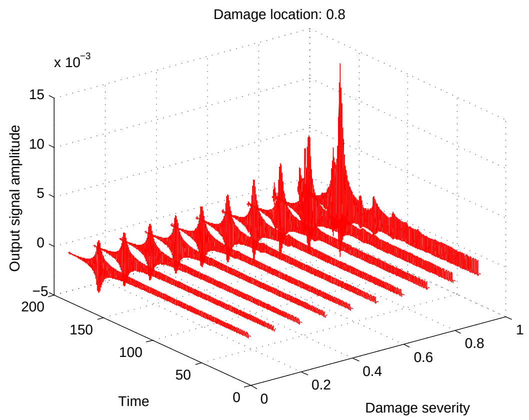

Fig. 4. Output signal amplitude for increasing damage severity - damage location is set to 0.8

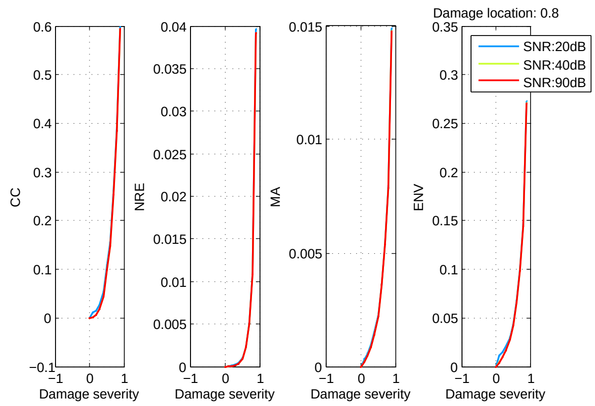

Fig. 5. Signal based features as function of damage severity - damage location is set to 0.8

monotonous variation. But this poses a priori no problem in terms of classification.

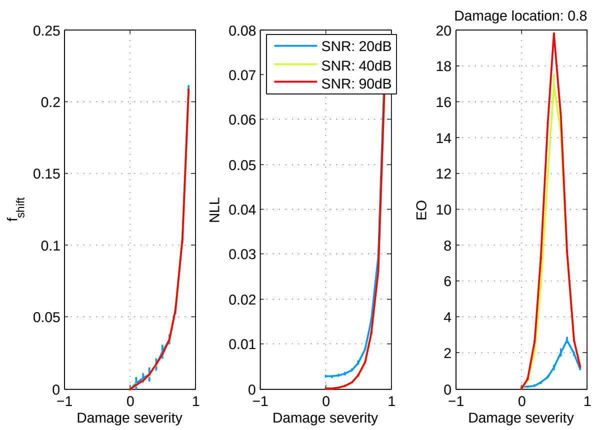

Fig. 6. Nonlinear model based features as function of damage severity - damage location is set to 0.8

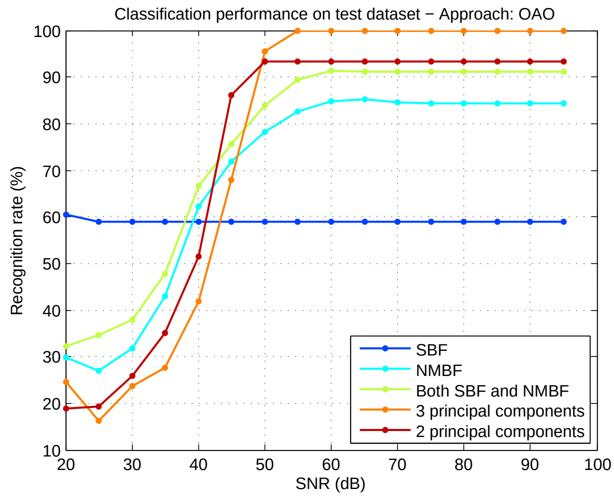

Fig. 7. Comparison of classification performance for various features scenarios - OAO approach

5.4 Classification performances

Figure 7 illustrates the recognition rate of various 'OAO' multiclass classifiers on test data (in %) versus SNR (in dB) contained in output signals. It can be seen that for high signal to noise ratios (namely SNR greater than 60 dB), the best classifier, in terms of test data recognition rate, is the one which was trained using the first three principal components obtained after performing PCA on both SBF and NMBF. Then comes the classifier trained with the first two principal components. The classifier trained on both SBF and NMBF arrives third in terms of recognition rate on test data while NMBF trained classifier comes fourth. Finally, comes SBF trained classifier. For low signal to noise ratios (lower than 40 dB), SBF trained classifier performs better than all the other classifiers.

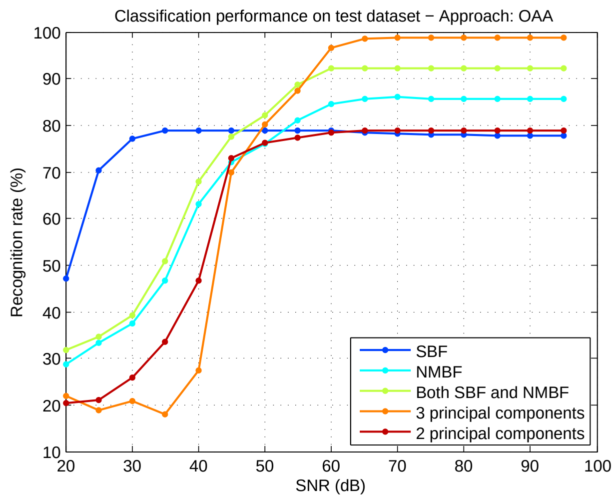

Fig. 8. Comparison of classification performance for various features scenarios - OAA approach

Figure 8 illustrates the recognition rate of various 'OAA' multiclass classifiers on test data (in %) versus SNR (in dB) contained in output signals. It can be seen that for high signal to noise ratios (namely SNR greater than 50 dB), the best classifier, in terms of test data recognition rate, is the one which was trained using the first three principal components obtained after performing PCA on both SBF and NMBF. Then comes the classifier trained with both SBF and NMBF. The classifier trained on only NMBF arrives third in terms of recognition rate on test data. Finally, SBF trained classifier as well as the classifier trained on the first two principal components obtained after performing PCA on both SBF and NMBF, have approximately the same recognition rate on test data. For low signal to noise ratios (lower than 50 dB), SBF trained classifier performs better than all the other classifiers.

6. CONCLUSION AND PERSPECTIVES

From the outcome of our investigation it is possible to conclude that:

- For high values of SNR, NMBF bring forward more information on damage severity. Thus by introducing such features within the inputs of the SVM classifiers, classification performances are significantly improved. This applies to both 'OAO' and 'OAA' approaches.
- For high values of SNR and by performing PCA on both SBF and NMBF, classification performances are improved. Thus, PCA allows getting higher recognition rates on test data while reducing the dimension of features vector.
- Classifiers trained on NMBF or on Principal components are more sensitive to output signals noise than the classifiers trained on SBF.
- For low SNR values, Classifiers trained on principal components are the most degraded in terms of test data recognition rate. This underlines one limitation of calssic PCA which does not distinguish between variance due to measurement noise and variance due to genuine underlying signal variations.

In our future research we intend to:

- Apply the quantification approach proposed in this work to a beam model with other types of nonlinearities such as gap, jump or saturation nonlinearities.
- Apply the quantification approach proposed in this work to a plate like model with a delamination-type damage.
- Use a probabilistic Support Vector Classification.
- Test the quantification workflow presented in this paper on real test structures equipped with piezoelectric elements.

REFERENCES

Bakir, M., Rébillat, M., and Mechbal, N. (2015). Damage type classification based on structures nonlinear dynamical signature. In *9th IFAC symposium on Fault Detection, Supervision and Safety of Technical Processes*, 652–657. Paris.

Balmès, E. (2016). Sdtools, vibration software and consulting. `http://www.sdtools.com/`.

Boser, B.E., Guyon, I.M., and Vapnik, V.N. (1992). A training algorithm for optimal margin classifiers. In *Proceedings of the Fifth Annual Workshop on Computational Learning Theory*, COLT '92, 144–152. ACM, New York, NY, USA.

Canu, S., Grandvalet, Y., Guigue, V., and Rakotomamonjy, A. (2005). Svm and kernel methods matlab toolbox. `http://asi.insa-rouen.fr`.

Coverley, P.T. and Staszewski, W.J. (2003). Impact damage location in composite structures using optimized sensor triangulation procedure. *Smart Materials and Structures*, 12, 795–803.

Cristianini, N. and Shawe-Taylor, J. (2000). *An Introduction to Support Vector Machines: And Other Kernel-based Learning Methods*. Cambridge University Press.

Fendzi, C. (2015). *Contrôle Santé des Structures Composites : Application à la Surveillance des Nacelles Aéronautiques*. Ph.D. thesis, Arts et Métiers ParisTech.

Fendzi, C., Mechbal, N., Rébillat, M., and Guskov, M. (2016). A General Bayesian Framework for Ellipse-based and Hyperbola-based Damage Localisation in Anisotropic Composite Plates. *Journal of intelligent material sytstems ans structures*, 27(3), 350–374.

Hajrya, R. and Mechbal, N. (2013). Principal component analysis and perturbation theory-based robust damage detection of multifunctional aircraft structure. *Structural Health Monitoring*, 12(3), 263–277.

Jolliffe, I. (1986). *Principal Component Analysis*. Springer Verlag.

Kim, D. and Philen, M. (2011). Damage classification using Adaboost machine learning for structural health monitoring. In *Proc. SPIE*, 1226–2013.

Mao, Z. and Todd, M. (2014). Structural Damage Classification Comparison Using Support Vector Machine and Bayesian Model Selection. In *7th European Workshop on Structural Health Monitoring*, 1973–1980.

Rébillat, M., Barthes, C.B., Mechbal, N., and Mosalam, K.M. (2014). Structural health monitoring of high voltage electrical swich ceramic insulators in seismic areas. In *7th European Workshop on Structural Health Monitoring*, 2183–2190. Nantes.

Rébillat, M., Ege, K., Gallo, M., and Antoni, J. (2016). Repeated exponential sine sweeps for the autonomous estimation of nonlinearities and bootstrap assessment of uncertainties. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 230(6), 1007–1018.

Rébillat, M., Hajrya, R., and Mechbal, N. (2013). Detection of structural damage using the exponential sine sweep method. In *International Workshop on Structural Health Monitoring*, 1226–2013.

Rytter, A. (1993). *Vibrational Based Inspection of Civil Engineering Structures*. Ph.D. thesis, Aalborg University.

Tibaduiza, D., Mujica, L., and Rodellar, J. (2013). Damage classification in structural health monitoring using principal component analysis and self-organizing maps. *Structural Control and Health Monitoring*, 1303–1316.

Vapnik, V.N. (1998). *Statistical Learning Theory*. Wiley-Interscience.

Vergé, M., Mechbal, N., and Hajrya, R. (2010). Active Damage Detection and Localization Applied to Composite Structure Using Piezoceramic Patches. In *Conference on Control and Fault Tolerant Systems*, 1. Nice, France. 8 pages.

Vitola, J., Tibaduiza, D., Anaya, M., and Pozo, F. (2016). Structural Damage detection and classification based on Machine learning algorithms. In *8th European Workshop On Structural Health Monitoring*, July, 5–8.

Worden, K., Farrar, C.R., Haywood, J., and Todd, M. (2008). A review of nonlinear dynamics applications to structural health monitoring. *Structural Control and Health Monitoring*, 15, 540–567.